Dipole Cross Section in DIS

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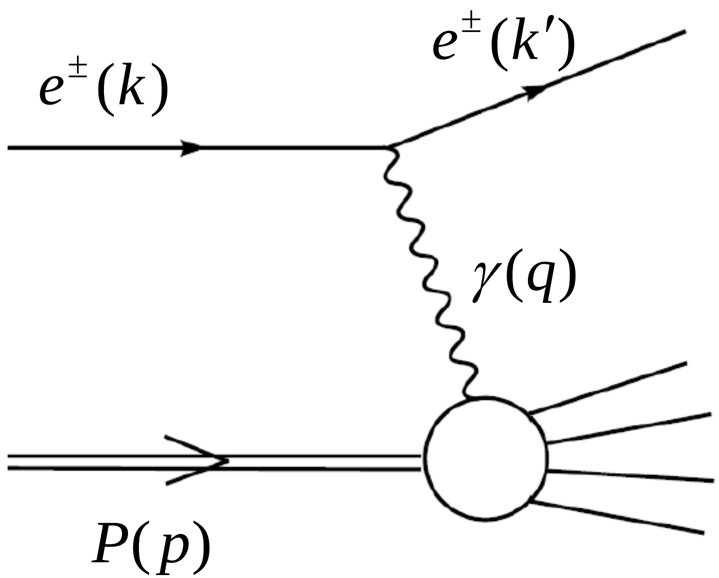
Outline

- Introduction
  - DIS/Saturation/Color Dipole Model
- Formalism
- Results (preliminary)
  - Dipole cross section
  - F2-comparison
- Conclude

Deep Inelastic Scattering



$$Q^2 = -q^2$$

$$W^2 = (k+p)^2$$

$$x = \frac{Q^2}{2m_p \nu} = \frac{Q^2}{W^2 + Q^2 - m_P^2}$$

$$\nu = p \cdot q / m_P$$

$$\frac{d\sigma}{dxdQ^2} \propto \left(1 + (1-y)^2\right) F_2(x, Q^2) - y^2 F_L(x, Q^2)$$

$$F_2(x, Q^2) = \sum_i e_i^2 x f_i(x)$$

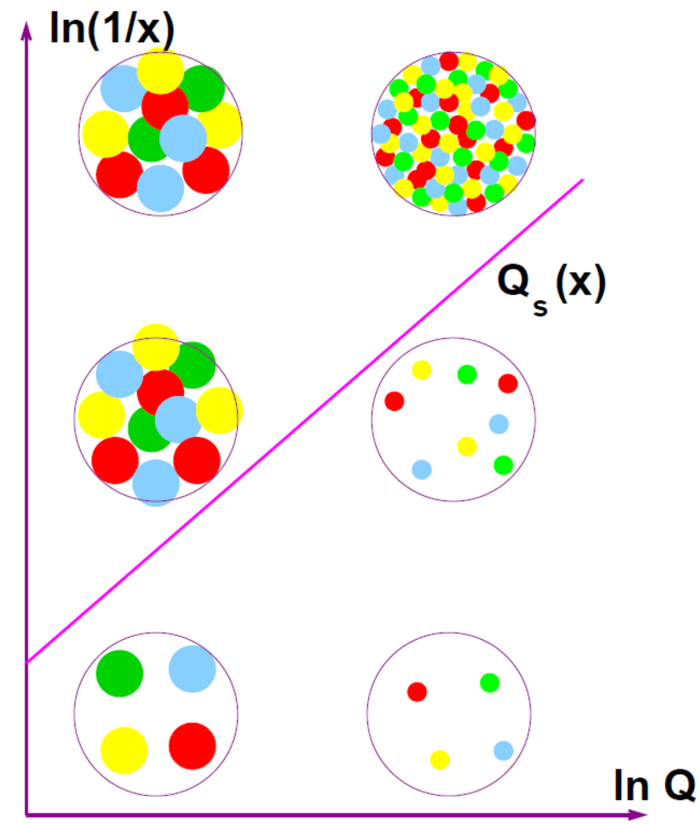
Saturation

$$W^2 >> Q^2 \qquad x \to \frac{Q^2}{W^2}$$

At very small x, gluon density becomes increased and gluons can be recombined.
→ Saturation

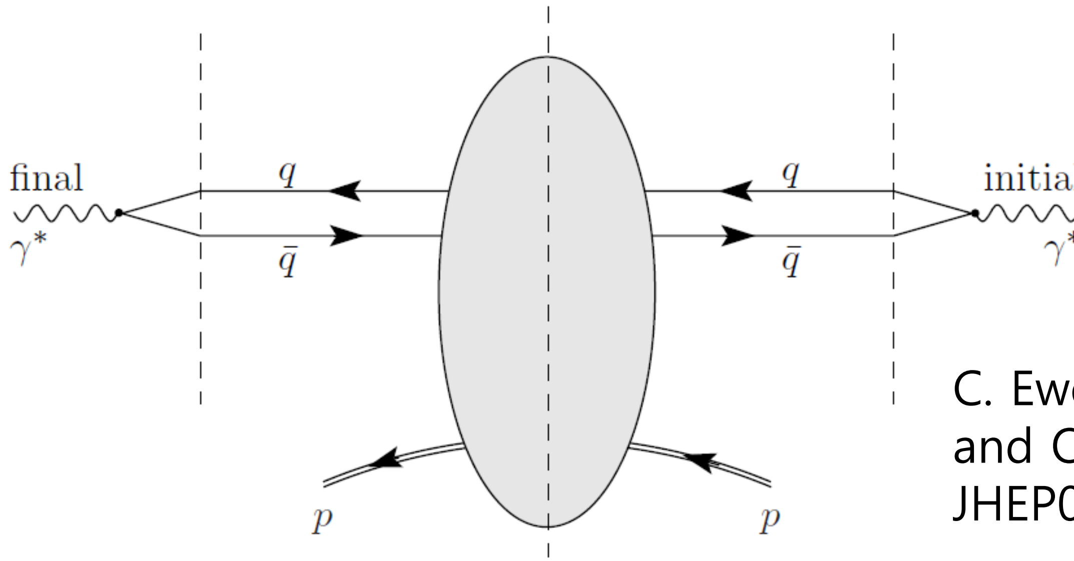
DGLAP does not hold at small x region.

As an alternative way, the dipole model was suggested.



K.Golec-Biernat – conf. proceeding
arXiv:0812.1523

Color Dipole Model



C. Ewerz, A. Manteuffel
and O. Nachtmann
JHEP03 (2011) 062

At high energy, photon splits into a quark-antiquark pair (color dipole) before scattering on the proton

$$F_2(x,Q^2)=\frac{Q^2}{4\pi^2\alpha_e}\left[\sigma_L(x,Q^2)+\sigma_T(x,Q^2)\right]$$

Color Dipole Model

$$\sigma_{L,T}(x,Q^2)=\sum_f \int d^2 r\,\omega_{T,L}^{(q)}(r,Q^2)\hat{\sigma}_d(r,x)$$

$$\omega_{T,L}^{(q)}(r,Q^2)=\int_0^1 d\alpha \left|\Psi_{T,L}^{(f)}(r,\alpha,Q^2)\right|^2$$

$$\left|\Psi_T^{(f)}(r,\alpha,Q^2)\right|^2=e_f^2\,\frac{\alpha_e Nc}{2\pi^2}\left(\left(\alpha^2+(1-\alpha)^2\right)\overline{Q}_f^2 K_1^2(r\overline{Q}_f)+m_f^2 K_0^2(r\overline{Q}_f)\right)$$

$$\left|\Psi_L^{(f)}(r,\alpha,Q^2)\right|^2=e_f^2\,\frac{\alpha_e Nc}{2\pi^2}4Q^2\alpha^2(1-\alpha)^2 K_0^2(r\overline{Q}_f)$$

$$\overline{Q}_f^2=\alpha\left(1-\alpha\right)Q^2+m_f^2$$

C. Ewerz, A. Manteuffel, and O. Nachtmann JHEP03 (2011) 06

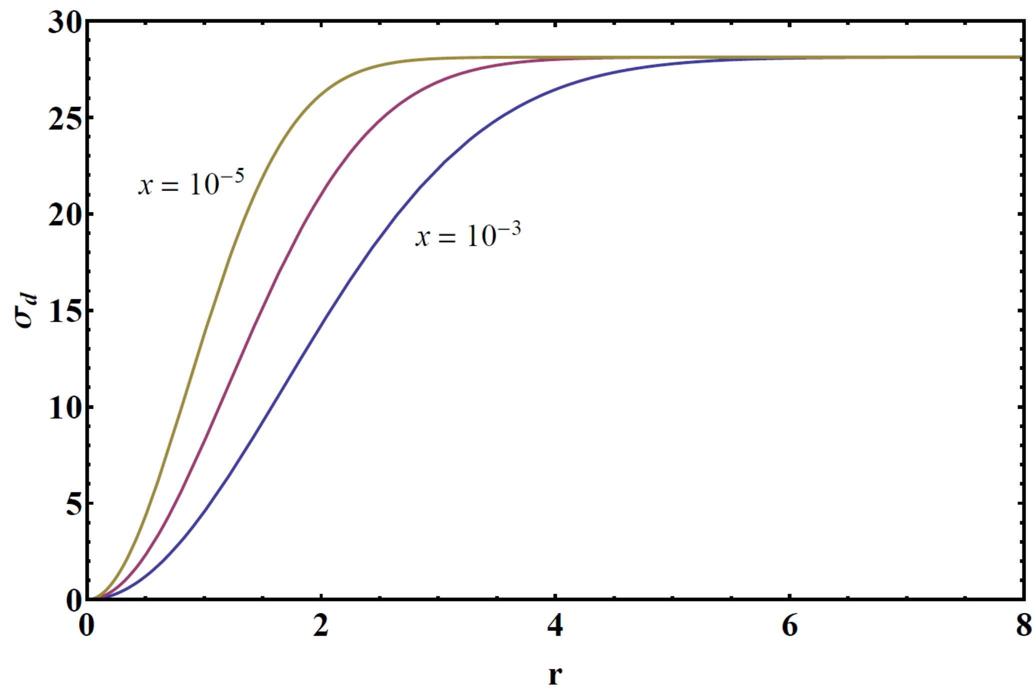
GBW Model

Golec-Biernat and Wüsthoff (GBW) model includes the saturation and the geometric scaling.

$$\sigma_d = \sigma_0 \left(1 - e^{-r^2/4R_0^2}\right)$$

$$R_0^2 = \left(x/x_0\right)^{\lambda}$$

R_0 : saturation scale



GBW – Golec-Biernat & Wusthoff, Phys. Rev. D 59 (1998) 014017

Formalism

Dipole Formula (Ewerz)

$$F_2(x,Q^2) = \sum_f Q \int dr h(Qr, m_f r) \frac{1}{r^2} \hat{\sigma}(r,x)$$

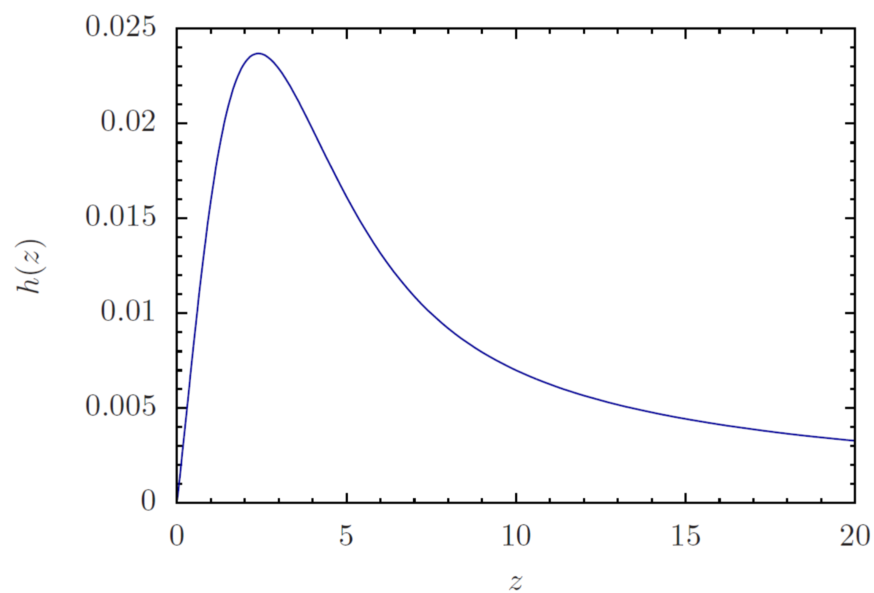
$$h(Qr, m_f r) = \frac{Qr^3}{2\pi\alpha_{em}} \int_0^1 d\alpha \left[\left|\Psi_T(r,\alpha,Q^2)\right|^2 + \left|\Psi_L(r,\alpha,Q^2)\right|^2 \right]$$

When mf=0,

$$h(Qr, m_f r) \to h(z)$$

$$z \equiv Qr$$

$$z_0 = 2.4$$



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Dipole Formula (Ewerz, m_f=0)

$$t = \ln\left(Q / Q_0\right) \qquad t' = -\ln\left(r / r_0\right)$$

$$F_2(x, Q_0^2 e^{2t}) = \sum_f z_0 e^t \int_{-\infty}^{\infty} dt' h(z_0 e^{t-t'}) \frac{1}{r^2} \hat{\sigma}(r, x)$$

$$S(x, t') \equiv \frac{1}{r r_0} \hat{\sigma}_0(r, x)\Big|_{r = r_0 \exp(-t')}$$

$$F_2\left(x, Q_0^2 e^{2t}\right) e^{-t} = \int_{-\infty}^{\infty} dt' \kappa(t - t') S(x, t')$$

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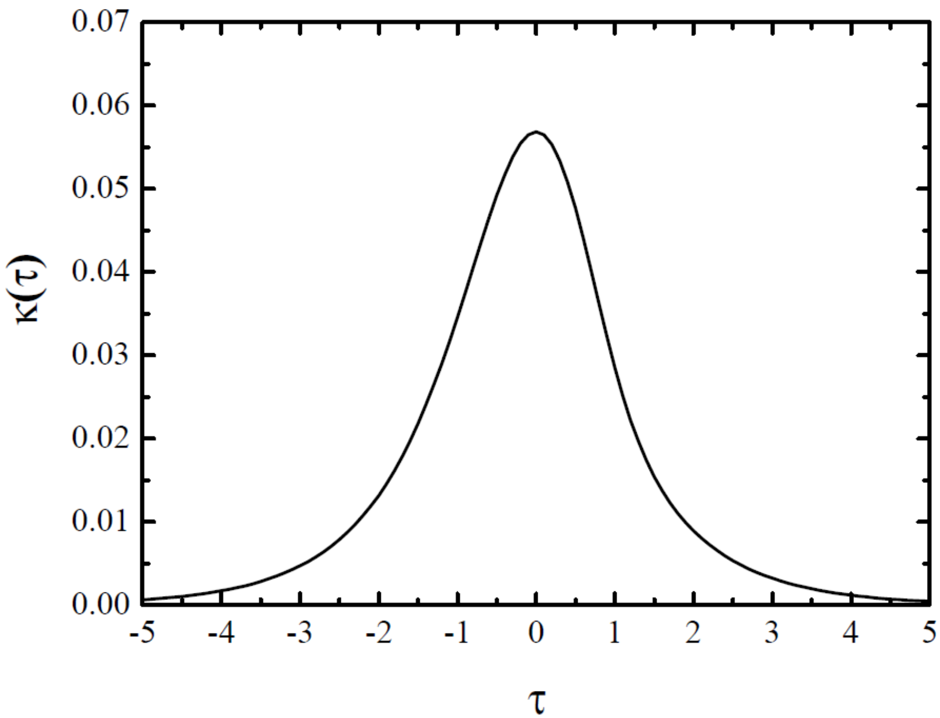
Dipole Formula (Ewerz,m_f=0)

$$t - t' \equiv \tau = \ln(z / z_0)$$

$$\kappa(\tau) \equiv z_0 h\left(z_0 e^{\tau}, 0\right)$$

κ(τ) are almost symmetric at τ=0 .



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Fourier Integral

$$F(x,t) \equiv F_2\left(x, Q_0^2 e^{2t}\right)e^{-t} = \int_{-\infty}^{\infty} dt' \kappa(t-t') S(x,t')$$

$$f(t) = \int_0^{\infty} dk \left\{a_f(k) \cos kt + b_f(k) \sin kt\right\}$$

$$a_f(k) = \frac{1}{\pi} \int_{-\infty}^{\infty} dt \cos kt\, f(t)$$

$$b_f(k) = \frac{1}{\pi} \int_{-\infty}^{\infty} dt \sin kt\, f(t)$$

$$\pi a_F(k) = \int_{-\infty}^{\infty} dt \cos kt \int_{-\infty}^{\infty} dt' \kappa(t-t') S(t') \qquad \tau = t - t'$$

$$= \int_{-\infty}^{\infty} d\tau dt' \left[\cos k\tau \cos kt' - \sin k\tau \sin kt'\right] \kappa(\tau) S(t')$$

$$= \pi a_\kappa(k) \pi a_S(k) - \pi b_\kappa(k) \pi b_S(k)$$

$$\approx \pi a_\kappa(k) \pi a_S(k)$$

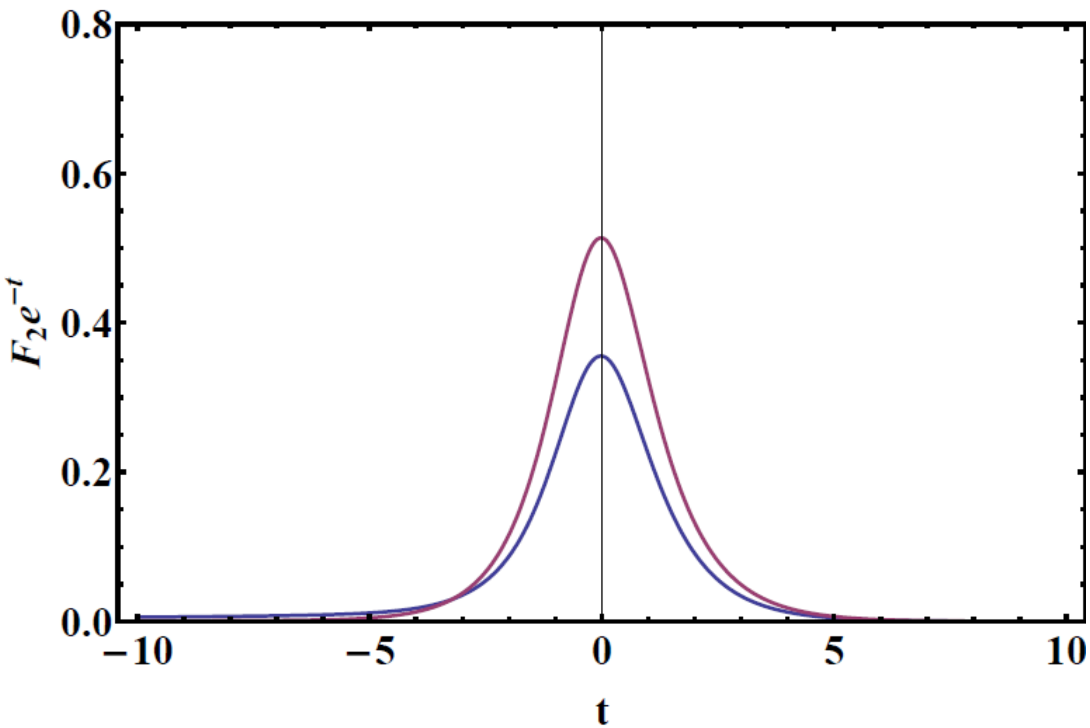
Approximation: $b_\kappa \approx 0$

$$\pi b_F(k) = \int_{-\infty}^{\infty} dt \sin kt \int_{-\infty}^{\infty} dt' \kappa(t-t') S(t')$$

$$\approx \pi a_\kappa(k) \pi b_S(k)$$

Structure Function F_2

$$F_2(x,Q^2) = Ax^{1-\alpha}\left(\frac{Q^2}{Q^2+a}\right)^{\alpha} + Bx^{1-\beta}\left(\frac{Q^2}{Q^2+b}\right)^{\beta}$$



$A = 0.324$

$B = 0.098$

$a = 0.5616$

$b = 0.01114$

$\alpha = 1.0808$

$\beta = 0.5475$

A. Donnachie, and P. V. Ladshoff - Z. Phys. C61 139 (1994)

The dipole cross section (m_q=0)

$$a_S(k) = \frac{1}{\pi}\frac{a_F(k)}{a_\kappa(k)} \qquad b_S(k) = \frac{1}{\pi}\frac{b_F(k)}{a_\kappa(k)}$$

$$a_F(k) = \frac{1}{\pi}\int_{-\infty}^{\infty} dt F_2(x, Q_0^2 e^{2t}) e^{-t}\cos kt$$ (Sin kt for $b_F(k)$)

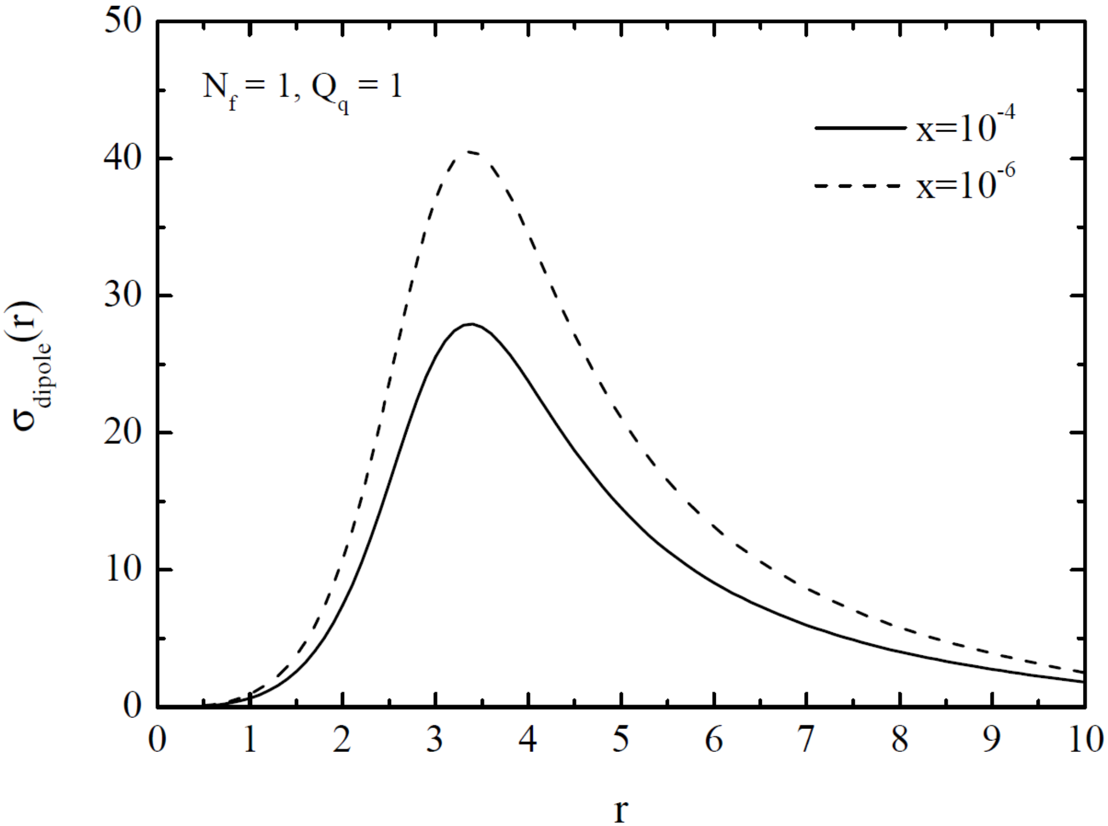
$$a_\kappa(k) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\tau \kappa(\tau)\cos k\tau$$

$$S(t') = \int_0^\infty dk\left\{a_S(k)\cos kt' + b_S(k)\sin kt'\right\}$$

$$\hat{\sigma}_0(x, r) = r r_0 S(x, t')$$

Results

The dipole cross section (m_q=0)



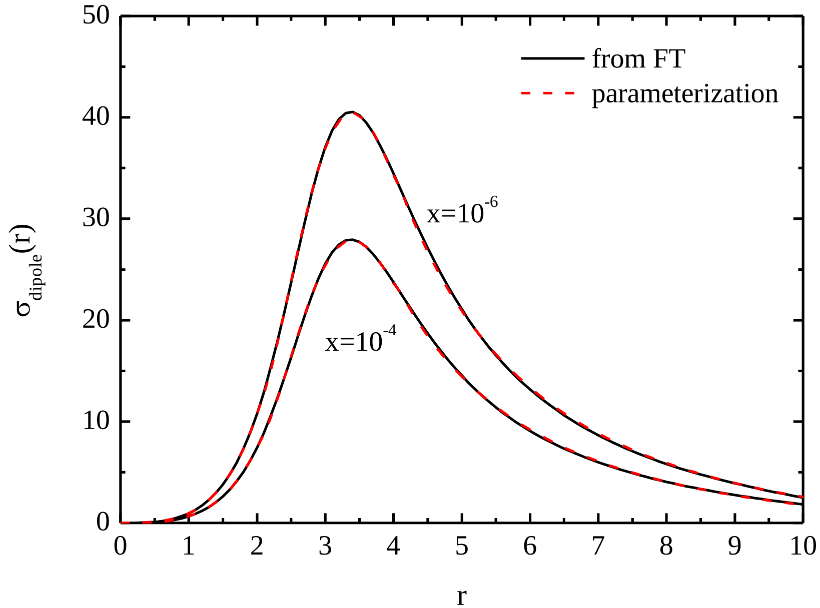
$$\hat{\sigma}(r,x) = \hat{\sigma}_0 \left(\frac{x}{10^{-4}} \right)^{-0.0808}$$

$$\hat{\sigma}_0 = \hat{\sigma}(r, x = 10^{-4})$$

Parameterization

$$\hat{\sigma}_0 = s_0 \left(\frac{x}{10^{-4}} \right)^{1-\alpha} \left(1 - \exp\left(a r^b \right) + c r^d \right)$$

$$\hat{\sigma}_0 = s_0 \left(\frac{x}{10^{-4}} \right)^{1-\alpha} \times \left(a + br + cr^2 + dr^3 + er^4 \right)$$

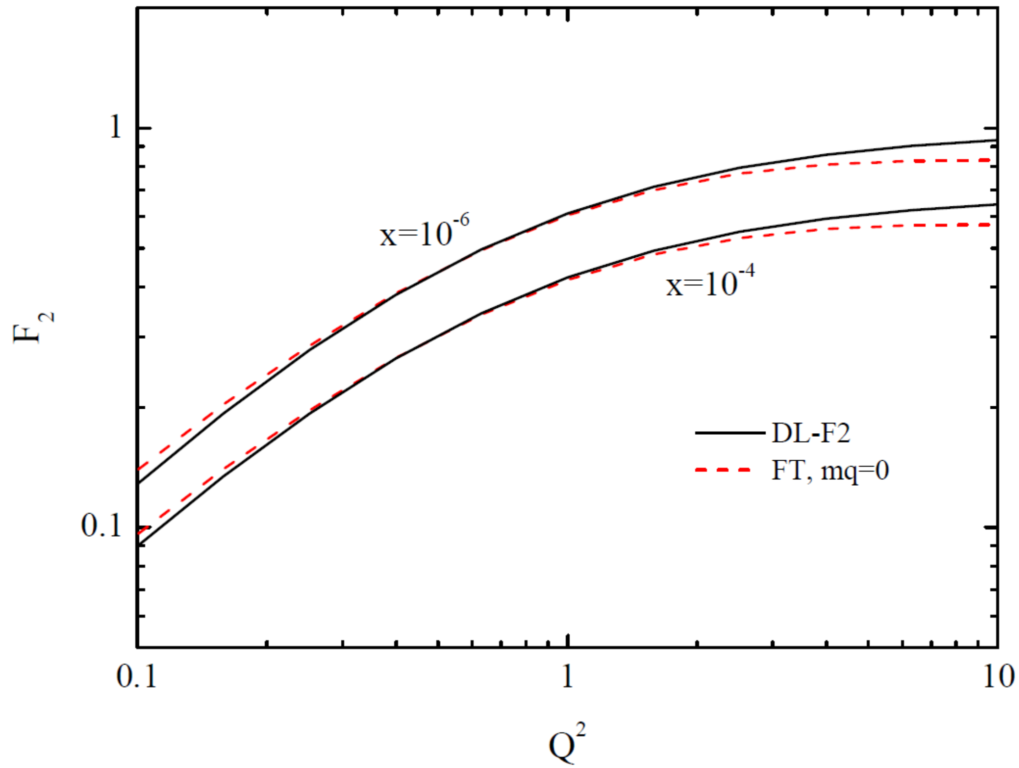


r-range	a	b	c	d	e
$r \leq 2$	-1.794×10^{-3}	5.373	2.127×10^{-2}	3.193	-
$2 < r \leq 4$	13.627	-20.066	10.613	-2.32	0.1803
$r > 4$	1.051	-0.796	15.392	-1.796	-

Preliminary

F_2 from σ_{dipole}

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_e}\left[\sigma_L(x,Q^2) + \sigma_T(x,Q^2)\right]$$



$$\sigma_{L,T}(x,Q^2)$$

$$= \sum_f \int d^2 r \omega_{T,L}^{(q)}(r,Q^2)\hat{\sigma}(r,x)$$

σ with massive quarks

- When $W^2 \cong Q^2/x$, at small x W^2 is greater than m_b^2. All 5-flavors are participated in the interaction.

- The dipole cross section for massive quarks can be determined with the normalization factor N_{σ}.

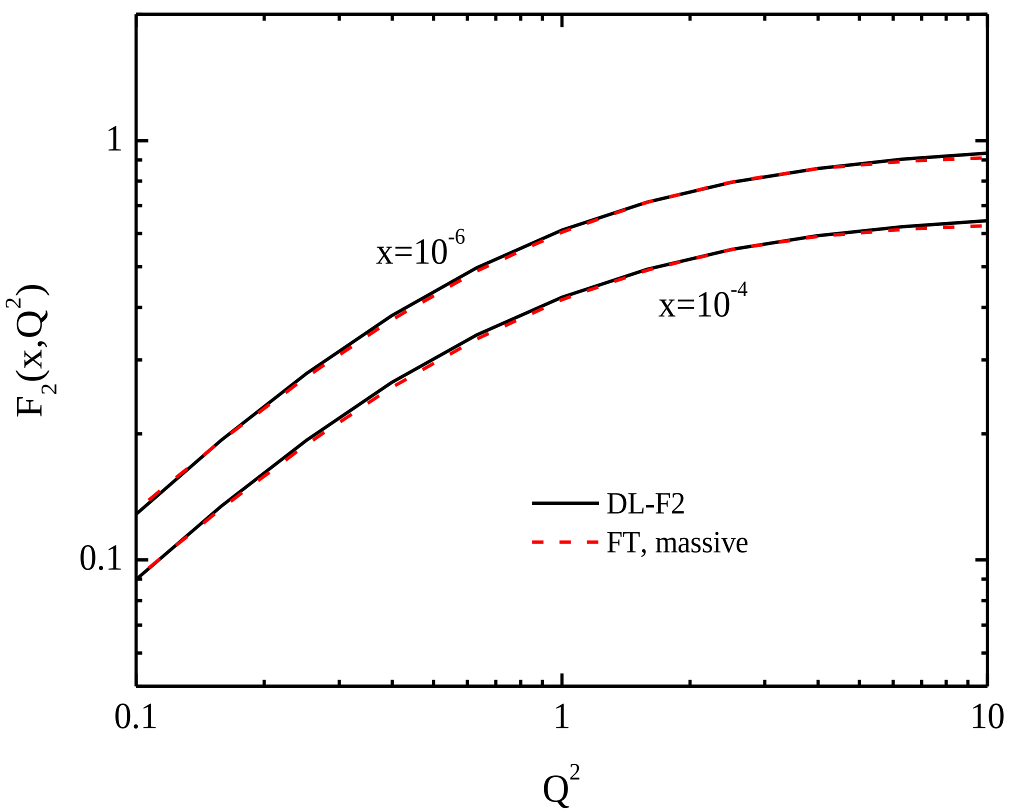
$$\hat{\sigma} = N_{\sigma}\hat{\sigma}_0 \cong 1.571\hat{\sigma}_0$$

$$N_{\sigma}^{-1} = (Q_u^2 + Q_d^2 + Q_s^2)\eta_{m_{\{u,d,s\}}} + Q_c^2\eta_{m_c} + Q_b^2\eta_{m_b}$$

$$\eta_{m_{\{u,d,s\}}} = 0.9292 \qquad \eta_{m_c} = 0.0376 \qquad \eta_{m_b} = 7.959 \times 10^{-4}$$

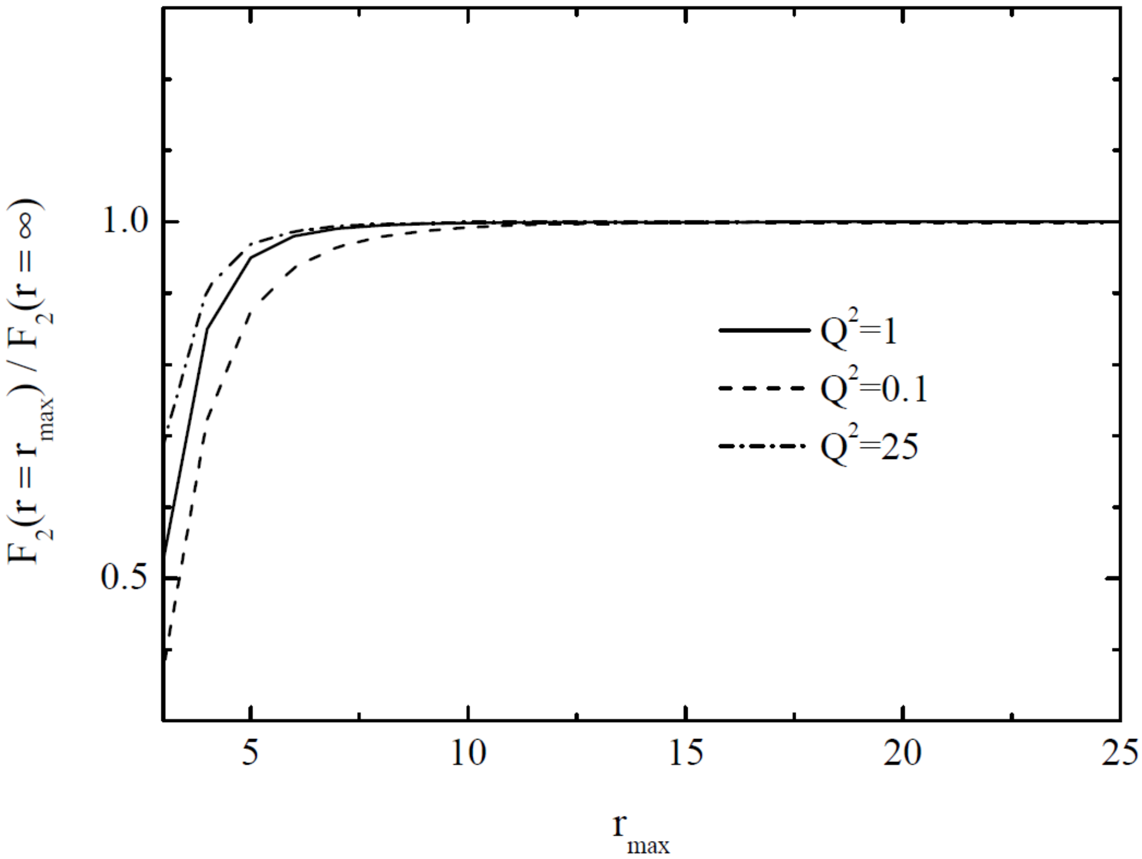
- η-values indicate the quark mass effects on the photon wave functions

F_2 from σ_{dipole} (massive)



- The dipole cross section with the mass and charge of quarks gives a better fit to original parameterizations of F_2.

Range of r for F_2



The range of r needed to evaluate F2 DL.

Conclusion

- Existing literature:
  - Theoretical form of dipole cross section, fit parameters to match measure F_2
  - Theoretical forms of F_2 to fit data
  - Approximate connections between F_2 and dipole cross section for large and small Q (Ewerz et al)
- New here:
  - we have inverted the relation between F_2 and dipole cross section directly for a simple parameterization of F_2 (Donnachie-Landshoff)

- We find that consistent with Ewerz et al discussion, unless the perturbative photon wave function is modified for low Q, the dipole cross section should fall as r becomes large (not constant cross section).

- Ewerz et al derive approximate low r relationship between F_2 and dipole cross section. Our result is intermediate between their relation and the GBW relation with sigma scaling like r^2.

- Work in progress: would like to get a better understanding to get a tighter relation between parameterizations of F_2 and parameterizations of the dipole cross section.